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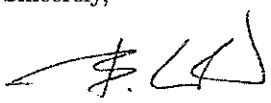
Report on the thesis of Jan Lastovicka

Dear colleague,

on the following pages you find my report on the Ph.D. thesis submitted by Mgr. Jan Lastovicka. My general impression is very positive. I see a professionally written mathematical text with potential impact in Informatics. The work contains original results of interest which have enough substance to be published in international peer reviewed journals. I therefore strongly support awarding the doctoral degree to Mgr. Lastovicka.
I congratulate your faculty on having so well educated graduate students.

Sincerely,



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Report on the thesis entitled
A model of sets with incomplete information
submitted by Mgr. Jan Lastovicka

The work presented on close to 90 pages by Mgr. Lastovicka unfolds a mathematical theory for dealing with incomplete information. It presents work that was developed over several years by the authors (together with his supervisor), and which has been discussed on several occasions and in preliminary publications. Now, with the dissertation, we see a systematically built theory fromits basic definitions and motivations to more advanced structural results.

About the reviewer: he is a full professor of mathematics („algebraic theory of structures") in active retirement („Seniorprofessor") at Technische Universität Dresden, Germany. In his carreer he has been the supervisor of 18 successful Ph.D. projects. His main scientific contributions are to a field called „Formal Concept Analysis", a mathematical theory of concepts and concept hierarchies, mostly applied in Computer Science. It is said that a strength of this field, in contrast to some other methods used for data analysis and knowledge processing, is its broad and elaborate mathematical foundation.

This is similar for the work presented in the dissertation of Mgr. Lastovicka. Although it developes in its essence a mathematical framework, the orientation and motivation lies in its potential to treat knowledge with incomplete informations. The carefully developed mathematical grounds may lead to much more powerful tools than an ad hoc treatment of such data ever could. This justifies, in the eyes of the reviewer, that this dissertation is submitted for the field of Informatics.

It comes at no surprise that a contribution like this one comes from Palacky University, which is renowned for its strong group in the field of fuzzy logic. The reviewer is not knowledgeable enough to judge how much of Mgr. Lastovicka's theory is already present in fuzzy theory. The ideas are related, but the flavour is clearly different.

The author's way of attacking the incompleteness of knowledge is that he assigns „conditions", under which certain information is available or not. He does that in a fundamentally systematic way, starting with a universe, then building a set theory with subsets, mappings, relations etc., on top of which he can introduce richer structures. This makes his approach universal. He thereby has laid the grounds to introduce conditional structures wherever they are needed. He could e.g. define conditions algebras or conditional topological spaces, to name two possibilities. But that is not what he is aiming at: his motivation is in the representation of knowledge, and he builds methodology to do that including conditions.

So the „incomplete information", which is mentioned in the title of the dissertation, is not simply the absence of information. Instead, a framework is developed that can express the availability of information under certain conditions. These conditions are elegantly modeled as the elements of a Boolean algebra, and a natural interpretation would be to understand these conditions as propositional terms over some variables. The basic definition is elegantly compact: the conditions are not directly imposed on the elements of the universe, but on the pairs of a conditional equivalence relation. The generality of this approach allows, for example, also a temporal interpretation, which is of interest.

It should be said that such a systematic approach requires an extensive machinery, a plentitude of notions and definitions. The field is new to the reviewer and he has not completely learnt all the details, but he understands that they are necessary for a solid theory. Chapter 1 presents many basic (but non-trivial) results to get this machinery started safely. These are the prerequisites for what is demonstrated later on.

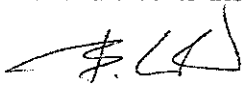
In the second chapter, a family of advanced structures is made conditional. The author can now build on what he has developed before. These structures are those of „Formal Concept Analysis", mentioned above as the main field of interest of the reviewer. His approach is smooth and natural, and the outcome is somewhat surprising. He proves the „basic theorem" (which always is the formidable task) for his setting, he can introduce the arrow relations, which are essential for the algebraic theory.
There are competing approachs to reach a similar task (describing conditional concepts). The most striking one is that of „Triadic Concept Analysis", where indeed conditional incidence relations are used, but in a slightly less structured way. These two approaches should lead to similar results, but they don't: While Lastovicka obtains a „conditional concept lattice", a triadic concept lattice is a completely different mathematical structure (a „trilattice"), which is difficult to handle and not fully understood. It is an unsolved task and an open problem to introduce arrow relations in triadic contexts. The reviewer would very much be interested to learn about the comparison of these two approaches during the defense.
This example shows that the work is not finished. There is still a lot to do. But the reviewer does not see that as a disadvantage, rather as inspiration. The theory presented by Mgr. Lastovicka invites to be applied and to be compared with other approaches. But it was necessary to work out the theory itself first.

About the quality and value of the presented results: The reviewer has no doubts that the submitted thesis is substantial enough for obtaining the degree of a Ph.D. (at least comparing to the standards in Germany). It clearly is a scientific contribution of importance, which can be published in peer reviewed journals (and parts of which already have been acceted for publication). The candidate emphazises his personal contributions, and the reviewer finds it credible that the dissertation contains much more than the already published work with his supervisor as co-author.

About form and diligence of the text: The dissertaion is professionally written, nicely presented in LaTeX, with clear language. It is easy to read, except for the mathematical difficulties, and with a few trivial exceptions, the reviewer did not find any mistakes or errors. Theorems and proofs could only be spot checked, but no problems accured. Again the reviewer concludes that the level of a Ph.D. thesis is clearly reached.

Critical remarks: The weakest ingredient of this work probably is its philosophical part. Nothing is wrong there, but perhaps more than a citation of a Plato dialogue is required to make this project a lasting success. The reviewer finds it (philosophically) risky to name one of the basic mappings a „reality", and he would prefer an even broader interface to other areas of thought. But one must admit that this is an unusual wish, which is very seldomly fulfilled in works on Mathematics or Informatics. Another criticism has already been mentioned: This work touches many other approaches, and thereby raises questions of comparison. This should be worked out in future research, but for the purpose of a Ph.D., the present version is substantial enough.

To conclude: Jan Lastovicka presents in the submitted thesis a well written mathematical contribution to Informatics, with many details of substantial difficulty. He shows with his original contributions, his style of presentation and his overall professional work that he is an state-of-the-art scientist and that he deserves the doctoral degree.



Prof. Dr. Bernhard Ganter

Report for PhD thesis of Jan Lastovicka

June 4, 2018

This report is based on the document entitled "A model of sets with incomplete information".

The main objective of the thesis is how to represent sets and relations with incomplete information in order to study their properties. The document is organized into two chapters :

The first chapter introduces incomplete information for sets and relations and a model to represent missing values using conditional equality and conditional membership. The missing information is represented using a boolean algebra of conditions. Then, a conditional set is modeled by a conditional equality (under which condition two elements are equal), and a conditional relation is modeled an incomplete set (a condition on the membership relation). The (partial) completion of missing information is then modeled using the notion of realization which corresponds to a mapping from a conditional universe L to another one K. Whenever $K = 2$, then we obtain an ordinary universe.

The second chapter studies incomplete information in discrete structures such as partially ordered sets and lattices. A conditional poset is defined as a conditional binary relation on a conditional set, where the notions of infimum and supremum are extended to conditional posets. Then the basic theorem for concept lattices is adapted to conditional concept lattices, where incomplete information comes from missing values in the context. Classical clarification and reduction are then extended to conditional context.

Finally, the document lists some perspectives: (1) to simplify proofs by using ordinary mappings as realizations instead of partial mappings, (2) to extend boolean algebra to Heyting algebra for conditions. (3) to represent a conditional concept lattice by a labeled Hasse diagram.

The problem of modeling incomplete information in data is important for processing data with missing values. This problem has been already considered in database theory, where an incomplete relation is represented by a conditional table (see the work of Imielinśki and Lipski[1984] and Grahne[1991]). Up to my knowledge, Jan Lastovicka was the first to consider a formal model to deal with missing values in context and thus in concept lattices. The notion of conditional sets and relations is similar to fuzzy sets and relations but has different semantics.

The thesis is well written even if the notations are complicated. The obtained results seem theoretically of interests to computer science community, but it looks difficult to implement in practice due to the size of conditions can be large while the number of missing values increases.

The thesis opens several questions related to a possible completion or all completions. For example, is there a completion for which a conditional concept lattice is distributive (the distributivity is possible), or, if the conditional concept lattice is distributive for every completion (the distributivity property is certain). The same questions can be applied to concepts. We can also be interested to structural properties of the set of all completions and check if it helps for simplifying the proofs?

Jan Lastovicka has already published partial results of his thesis in a conference of lattices theory (ICFCA) and I have no doubt that others results will be published in journal in his field.

In conclusion, the manuscript of Jan Lastovicka includes interesting results which seem promising and I believe that his work deserves to be awarded a Phd degree.

Lhouari Nourine
Professor in computer science at Clermont Auvergne university (France)